

Neutral top-pion and the rare top decays $t \rightarrow cl_i l_j$

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Abstract

We study the rare top decays $t \rightarrow cl_i l_j (l = \tau, \mu, \text{or } e)$ in the framework of topcolor-assisted technicolor($TC2$) models. We find that the neutral top-pion π_t^0 can produce significant contributions to these processes via the flavor changing couplings $\pi_t^0 \bar{t}c$ and $\pi_t^0 l_i l_j$. For the π_t^0 mass $m_{\pi_t} = 150 GeV$ and the parameter $\varepsilon = 0.08$, the branching ratio $Br(t \rightarrow c\tau\tau)$ can reach 7.1×10^{-7} . Taking into account the constraints of the present experimental limit of the process $\mu \rightarrow e\gamma$ on the free parameters of $TC2$ models, we find that the value of $Br(t \rightarrow c\tau\mu) \approx Br(t \rightarrow c\tau e)$ is in the range of $1.8 \times 10^{-10} \sim 1.7 \times 10^{-8}$.

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I. Introduction

It is well known that, in the standard model(SM), the flavor changing neutral current ($FCNC$) is absent at tree-level and at one-loop level they are GIM suppressed. The SM results of the rare decays, which are induced by $FCNC$, are very small and can not be detected in the current or future high energy collider experiments. Thus, rare decays provide a very sensitive probe of new physics beyond the SM . Detection of rare decays at visible levels by any of the future colliders would be instant evidence of new physics. Searching for rare decays is one of the major goals of the next generation of high energy collider experiments.

The large value of the top quark mass offers the possibility that it can be singled out to play a key role in probing the new physics beyond the SM . The properties of the top quark could reveal information regarding flavor physics, electroweak symmetry breaking(EW SB) mechanism, as well as new physics beyond the SM [1]. One of these consequences is that rare top decays can be used to detect new physics. Much of the theoretical activity involving rare top decays has occurred within some specific models beyond the SM [2]. It has been shown that the values of the branching ratios $Br(t \rightarrow cv)(v = g, \gamma, \text{ or } z)$ or $Br(t \rightarrow cvv)(v = g, \gamma, \text{ or } w)$ can indeed be enhanced by several orders of magnitude, which may be within the observable threshold of near-future-experiments. Recently, the rare top decays $t \rightarrow cl_i l_j (l = \tau, \mu, \text{ or } e)$ have been studied in the framework of the general two Higgs doublet model[3]. They have shown that the neutral Higgs bosons h^0 and A^0 can make the branching ratios $Br(t \rightarrow cl_i l_j)$ reach 1×10^{-8} .

The presence of the top-pions $\pi_t^{0,\pm}$ in low-energy spectrum is an inevitable feature of topcolor scenario[4]. These new particles have large Yukawa couplings to the third family quarks and can induce the tree-level FC couplings, which have significant contributions to the rare top decays $t \rightarrow c\gamma(g, z)$ and $t \rightarrow cww$ [5]. In this paper, we will study the contributions of the neutral top-pion π_t^0 to the rare top decays $t \rightarrow cl_i l_j$ in the context of topcolor-assisted technicolor($TC2$) models[6]. These rare decay processes can occur at the tree-level or one-loop level, which are induced by the FC couplings $\pi_t^0 \bar{t}c$ or $\pi_t^0 l_i l_j (i \neq j)$. Our numerical results show that the neutral top-pion π_t^0 can generate

significant contributions to these processes. In all of the parameter space, the values of the branching ratio $Br(t \rightarrow c\tau\tau)$ are larger than those of $Br(t \rightarrow c\mu\mu)$ or $Br(t \rightarrow cee)$. For $m_{\pi_t} = 150\text{GeV}$ and the parameter $\varepsilon=0.08$, we have $Br(t \rightarrow c\tau\tau) = 7.1 \times 10^{-7}$. Taking into account the constraints of the present experimental limit of the process $\mu \rightarrow e\gamma$ on the flavor mixing parameter K_{ij} , we further calculate the contributions of π_t^0 to the rare top decays $t \rightarrow c\tau\mu$ and $c\tau e$. We find that the branching ratio $Br(t \rightarrow c\tau\mu)$ is approximately equal to the branching ratio $Br(t \rightarrow c\tau e)$. In most of the parameter space, the value of $Br(t \rightarrow c\tau\mu)$ is in the range of $1.8 \times 10^{-10} \sim 1.7 \times 10^{-8}$.

The paper is organized as follows: In section 2, we will present the expressions of the branching ratios $Br(t \rightarrow cl_i l_j)$ contributed by the neutral top-pion π_t^0 . Our numerical results and conclusions are given in section 3.

II. The neutral top-pion π_t^0 and the branching ratios $Br(t \rightarrow cl_i l_j)$

For $TC2$ models[4,6], the underlying interactions, topcolor interactions, are assumed to be chiral critically strong at the scale about 1TeV and coupled preferentially to the third generation, and therefore do not possess GIM mechanism. The non-universal topcolor interactions result in the FC coupling vertices when one writes the interactions in the mass eigen basis. Thus, the top-pions can induce the new FC scalar coupling vertices. The couplings of the neutral top-pion π_t^0 to ordinary fermions, which are related to the rare top decays $t \rightarrow cl_i l_j$, can be written as [7,8]:

$$\frac{m_t}{\sqrt{2}F_t} \frac{\sqrt{\nu_w^2 - F_t^2}}{\nu_w} [K_{UR}^{tt} K_{UL}^{tt*} \bar{t} \gamma^5 t \pi_t^0 + K_{UR}^{tc} K_{UL}^{tt*} \bar{t} \gamma^5 t \pi_t^0] + \frac{m_l}{\sqrt{2}\nu_w} \bar{l} \gamma^5 l \pi_t^0 + \frac{m_\tau}{\sqrt{2}\nu_w} K_{\tau i} \bar{\tau} \gamma^5 l_i \pi_t^0. \quad (1)$$

Where $\nu_w = \nu/\sqrt{2} = 174\text{GeV}$ and $F_t \approx 50\text{GeV}$ is the top-pion decay constant, which can be estimated from the Pagels-stokar formula. K_{UL} and K_{UR} are rotation matrices that diagonalize the up-quark mass matrix M_U , i.e. $K_{UL}^+ M_U K_{UR} = M_U^{dia}$. To yield a realistic form of the CKM matrix V , it has been shown that their values can be taken as [7]:

$$K_{UL}^{tt} \approx 1, \quad K_{UR}^{tt} = 1 - \varepsilon, \quad K_{UR}^{tc} \leq \sqrt{2\varepsilon - \varepsilon^2}. \quad (2)$$

In the following calculation, we will take $K_{UR}^{tc} = \sqrt{2\varepsilon - \varepsilon^2}$ and take ε as a free parameter, which is assumed to be in the range of 0.01-0.1[4,6]. $l_i (i = 1, 2)$ is the first (second)

lepton $e(\mu)$, $K_{\tau i}$ is the flavor mixing factor, which is the free parameter. Certainly, there is also the FC scalar coupling $\pi_t^0 \bar{\mu} e$. However, the topcolor interactions only contact with the third generation fermions. The flavor mixing between the first and second generation fermions is very small, which can be ignored. Thus, we have assumed $K_{\mu e} \approx 0$.

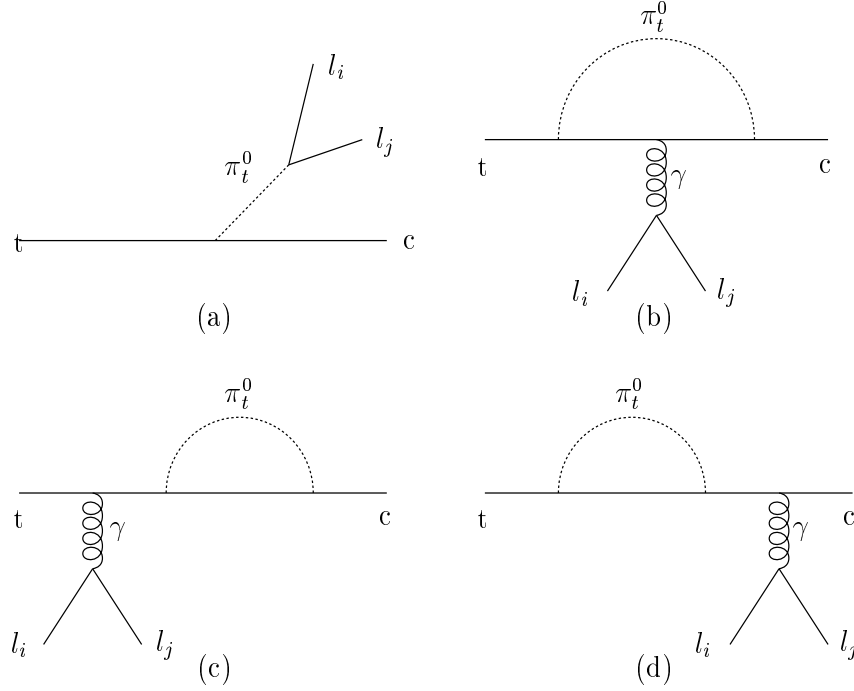


Figure 1: The tree-level and one-loop Feynman diagrams contribute to the rare top decays $t \rightarrow cl_i l_j$ induced by π_t^0 exchange in $TC2$ models.

In $TC2$ models, the rare top decays $t \rightarrow cl_i l_j$ can be induced at the tree-level and can also be induced via photon penguin diagrams at the one-loop level, as shown in Fig.1. For the diagrams Fig.1 (b), (c), and (d), we have taken $i=j$.

Let us first consider the contributions of the neutral top-pion π_t^0 to the rare top decays $t \rightarrow cl_l$ ($l = \tau, \mu, \text{ or } e$). Using Eq.(1), Eq.(2) and other relevant Feynman rules, the renormalized amplitude can be written as:

$$M_1 = M_{tree} + M_{loop}, \quad (3)$$

$$M_{tree} = A_1 \bar{u}_c \gamma_5 u_t \frac{i}{q^2 - m_{\pi_t}^2 + i m_{\pi_t} \Gamma_{total}} \bar{u}_l \gamma_5 v_l, \quad (4)$$

$$M_{loop} = \bar{u}_l(-ie\gamma_\nu)v_l \frac{-ig^{\mu\nu}}{p_\gamma^2 + i\varepsilon} \bar{u}_c \Lambda_{tc\gamma,\mu} u_t, \quad (5)$$

$$\Lambda_{tc\gamma,\mu} = A_2[\gamma_\mu F_1 + p_{t,\mu} F_2 + p_{c,\mu} F_3] \quad (6)$$

with

$$A_1 = i \frac{m_t}{\sqrt{2}F_t} \frac{\sqrt{\nu_w^2 - F_t^2}}{\nu_w} K_{UR}^{tc} K_{UL}^{tt*} \frac{m_l}{\nu}, \quad A_2 = \frac{i}{16\pi^2} \frac{em_t^2}{3F_t^2} \frac{\nu_w^2 - F_t^2}{\nu_w^2} K_{UR}^{tc} K_{UR}^{tt} (K_{UL}^{tt*})^2$$

$$F_1 = [(m_c - m_t)(-m_t C_{11} + m_t C_{12} + m_c C_{12}) - 2C_{24} + m_\pi^2 C_0 + B_0 - B_0^* - B_1'],$$

$$F_2 = 2m_c(C_{22} - C_{23}) + 2m_t(2C_{23} - C_{21} - C_{22}),$$

$$F_3 = -2(m_c - m_t)(C_{12} + C_{22}) - 2m_t C_{23}.$$

Where q is the four momenta of the neutral top pion π_t^0 and Γ_{total} is the total decay width of π_t^0 , which has been calculated in Ref.[9]. The expressions of the two-and three-point scalar integrals B_n and C_{ij} are [10]:

$$B_n = B_n(-\sqrt{\hat{s}}, m_t, m_t),$$

$$B_n^* = B_n(-P_c, m_{\pi_t}, m_t),$$

$$B'_n = B_n(-P_t, m_{\pi_t}, m_t),$$

$$C_{ij} = C_{ij}(p_t, -\sqrt{\hat{s}}, m_{\pi_t}, m_t, m_t),$$

$$C_0 = C_0(p_t, -\sqrt{\hat{s}}, m_{\pi_t}, m_t, m_t).$$

For the rare top decay processes $t \rightarrow c\tau l$ ($l = \mu$ or e), the neutral top-pion π_t^0 can only have contribution to these processes via Fig.1 (a). The renormalized amplitude can be written as:

$$M_2 = A_3 \bar{u}_c \gamma_5 u_t \frac{i}{q^2 - m_{\pi_t}^2 + im_{\pi_t} \Gamma_{total}} \bar{u}_\mu \gamma^5 v_\tau, \quad (7)$$

$$A_3 = i \frac{m_t}{\sqrt{2}F_t} \frac{\sqrt{\nu_w^2 - F_t^2}}{\nu_w} K_{UR}^{tc} K_{UL}^{tt*} \frac{m_\tau}{\nu} K_{\tau l}. \quad (8)$$

The decay width $\Gamma(t \rightarrow cl_i l_j)$ can be obtained in the usual way and it is given by

$$\Gamma(t \rightarrow cl_i l_j) = \frac{1}{2m_t} \int (2\pi)^4 \delta^4(p_t - p_c - p_{l_i} - p_{l_j}) \bar{\Sigma} |M|^2 \frac{d^3 p_c}{(2\pi)^3 2E_c} \frac{d^3 p_{l_i}}{(2\pi)^3 2E_{l_i}} \frac{d^3 p_{l_j}}{(2\pi)^3 2E_{l_j}}, \quad (9)$$

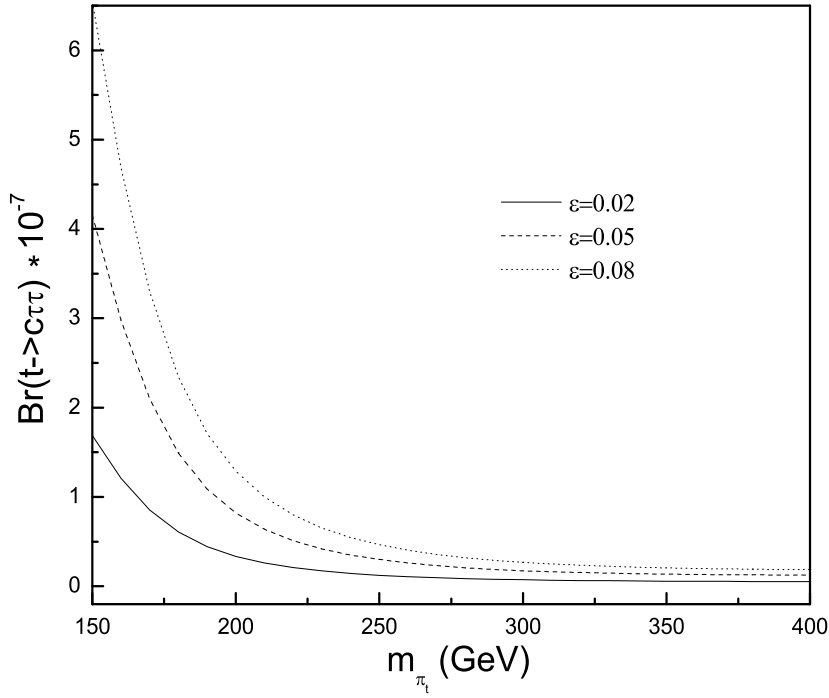


Figure 2: The branching ratio $Br(t \rightarrow c\tau\tau)$ as a function of the top-pion mass m_{π_t} for three values of the free parameter ε , in which we have included the tree-level and one-loop contributions of π_t^0 exchange.

where E_c , E_{l_i} , and E_{l_j} are the energies of the final particles c , l_i , and l_j , respectively. In the following section, we will use these formulae to give our numerical results.

III. Numerical results and conclusions

To calculate the branching ratios of the rare top decays $t \rightarrow cl_i l_j$, we assume that the total decay width Γ_t of the top quark is dominated by the decay channel $t \rightarrow wb$, which has been taken $\Gamma(t \rightarrow wb) = 1.56 GeV$. In our calculation, we have taken $m_t = 175 GeV$, $m_c = 1.2 GeV$, $m_\tau = 1.78 GeV$, $m_\mu = 0.105 GeV$, and $\alpha_e = \frac{1}{128.8}$ [11]. Except for these input parameters, there are three free parameters in the expressions of the branching ratios $Br(t \rightarrow cl_i l_j)$: ε , m_{π_t} and $K_{\tau l}$. To avoid significant fine-tuning, Ref.[6] has shown that the free parameter ε should be in the range of $0.01 \sim 0.1$. The limits on the mass m_{π_t} of the top-pion may be obtained via studying its effects on various experimental observables [4,12]. It has been shown that the m_{π_t} is allowed to be in the range of a few

hundred GeV depending on the models. As numerical estimation, we take the π_t^0 mass m_{π_t} to vary from $150GeV$ to $400GeV$. Our numerical results are summarized in Fig.2~Fig.4.

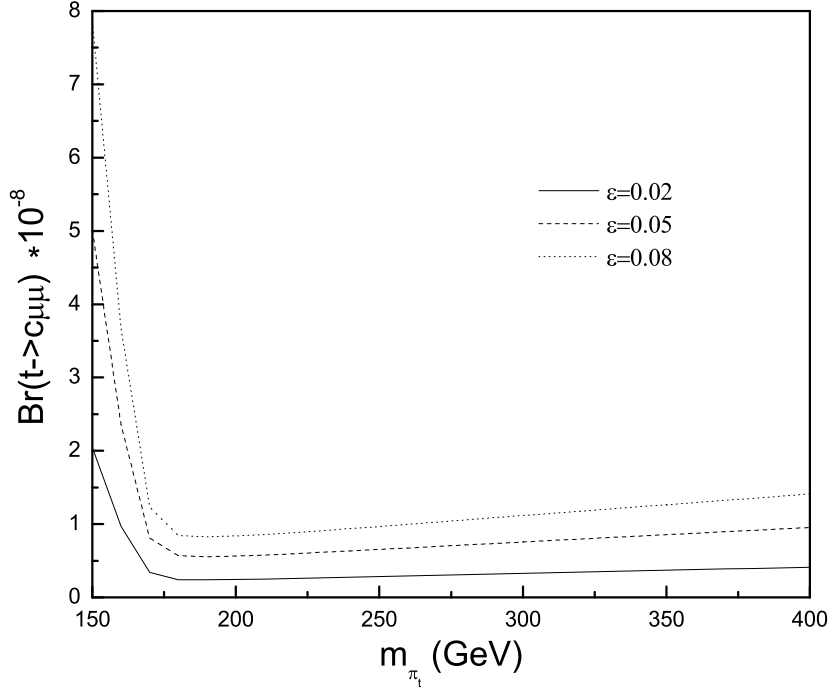


Figure 3: Same as Fig.2 but for the rare top decay $t \rightarrow c\mu\mu$.

The branching ratios $Br(t \rightarrow c\tau\tau)$ and $Br(t \rightarrow c\mu\mu)$ are plotted as functions of the π_t^0 mass m_{π_t} for three values of the free parameter ϵ in Fig.2 and Fig.3, respectively. From these figures, we can see that the values of these branching ratios increase as ϵ increasing. In all of the parameter space of the $TC2$ models, the values of branching ratio $Br(t \rightarrow c\tau\tau)$ are larger than those of $Br(t \rightarrow c\mu\mu)$. For example, for $m_{\pi_t} = 200GeV$ and $\epsilon = 0.05$, the values of $Br(t \rightarrow c\tau\tau)$ and $Br(t \rightarrow c\mu\mu)$ are 8.2×10^{-8} and 5.6×10^{-9} , respectively. This is because the one-loop contributions of π_t^0 exchange to the branching ratios $Br(t \rightarrow c\tau\tau)$, $Br(t \rightarrow c\mu\mu)$, and $Br(t \rightarrow cee)$ are approximately equal to each other, which are smaller than 5×10^{-9} . However, the tree-level contributions of π_t^0 exchange to the branching ratios $Br(t \rightarrow cll)$ are proportional to $\frac{m_t^2}{\nu}$. Thus, for the process $t \rightarrow c\tau\tau$, the tree-level contributions are larger than the one-loop contributions at least by two orders of

magnitude. For the process $t \rightarrow c\mu\mu$, two kinds of the contributions are comparable.

For the process $t \rightarrow cee$, the factor $\frac{m_e^2}{\nu^2}$ strongly suppresses the tree-level contributions, the values of $Br(t \rightarrow cee)$ is mainly generated by the one-loop contributions, which is smaller than 5×10^{-9} . Thus, if we would like to detect the possible signals of the neutral top-pion π_t^0 at the future high energy experiments via the rare top decays $t \rightarrow cll$, we should first consider the process $t \rightarrow c\tau\tau$. For instance, if we assume $m_{\pi_t} = 150\text{GeV}$ and $\varepsilon = 0.08$, then the values of the branching ratio $Br(t \rightarrow c\tau\tau)$ can reach 7.1×10^{-7} .

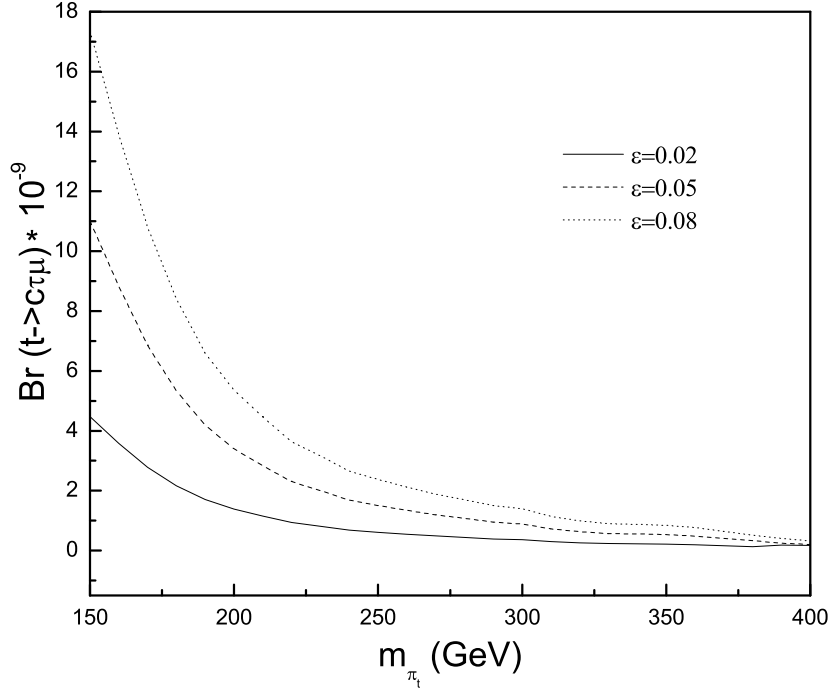


Figure 4: The branching ratio $Br(t \rightarrow c\tau\mu) \approx Br(t \rightarrow c\tau e)$ as a function of the π_t^0 mass m_{π_t} for three values of the free parameter ε .

From above discussions, we can see that the neutral top-pion π_t^0 can only produce the tree-level contributions to the rare top decays $t \rightarrow c\tau l$ ($l = \mu$ or e) via the Feynman diagram Fig.1(a). The expressions of the branching ratios $Br(t \rightarrow c\tau l)$ are dependent on the π_t^0 mass m_{π_t} , the flavor mixing factor $K_{\tau l}$, and the parameter ε . Furthermore, π_t^0 can produce significant contributions to the lepton flavor violating(LFV) processes $l_i \rightarrow l_j \gamma$ via the FC scalar couplings $\pi_t^0 \tau l$. For the LFV process $\mu \rightarrow e \gamma$, the neutral top-pion π_t^0

can contribute this process via the on-shell photon penguin diagrams generated by the FC scalar coupling $\pi_t^0 \tau \mu$ and $\pi_t^0 \tau e$. Thus, using the present experimental bound on the LFCV process $\mu \rightarrow e \gamma$, i.e. $Br(\mu \rightarrow e \gamma) \leq 1.1 \times 10^{-11}$, we can give the constraints on the flavor mixing factor K for $150 GeV \leq m_{\pi_t} \leq 400 GeV$, in which we have assumed $K = K_{\tau \mu} = K_{\tau e}$. If we take that m_{π_t} is smaller than $400 GeV$, then there must be $K \leq 0.2$ [8]. Taking into account the constraints on the flavor mixing factor K , the branching ratio $Br(t \rightarrow c \tau \mu) \approx Br(t \rightarrow c \tau e)$ is plotted as a function of m_{π_t} for three values of the parameter ε in Fig.4. One can see, from Fig.4, that the value of $Br(t \rightarrow c \tau \mu)$ increases as m_{π_t} decreasing and ε increasing. For $0.02 \leq \varepsilon \leq 0.08$ and $150 GeV \leq m_{\pi_t} \leq 400 GeV$, the value of $Br(t \rightarrow c \tau \mu) \approx Br(t \rightarrow c \tau e)$ is in the range of $1.8 \times 10^{-10} \sim 1.7 \times 10^{-8}$.

The *CERN LHC* will allow to probe the couplings of the top quark to both known and new particles involved in possible top decay channels different from the main $t \rightarrow wb$. It is very useful to analyze the rare top decays for detecting the new physics beyond the SM[1,2]. If we assume that the production cross section of the top quark pairs is about $8 \times 10^5 fb$ at the *LHC* with a yearly integrated luminosity of $\mathcal{L} = 100 fb^{-1}$, then we can only obtain 8 $c \tau \tau$ events per year for $m_{\pi_t} = 150 GeV$ and $\varepsilon = 0.08$.

The signal of the rare top decay $t \rightarrow c \tau \tau$ is the isolated τ leptons with a c -quark jet. If we want to detect this signal, we must consider the efficiency problems in measuring the τ lepton and identifying a c -quark jet. Furthermore, we will have to take the suitable kinematical cuts to extract the signal from a possible large reducible background, which will further degrade the signal. Thus, it is very difficult to detect the possible signals of the neutral top-pion π_t^0 by the process $t \rightarrow c \tau \tau$ at the future *LHC* experiments. However, if the topcolor scenario or other new physics beyond the SM can predict the existence of a light neutral scalar, for example $m_s < 100 GeV$, which can induce the FC scalar couplings, its signals might be detected via the process $t \rightarrow c \tau \tau$ at the future *LHC* experiments. Even if we can not observe the rare top decay $t \rightarrow c \tau \tau$ induced by the light neutral scalar, we can obtain experimental bounds on the masses of this kind of new particles[14].

The possibility of detecting the top-pions via the FC processes at the *LHC* experiments has been extensively studied in Ref.[7,13]. They have shown that the signals of

top-pions should be observable at the *LHC* in a sizable region of the parameter space. Therefore, we have to say that the top-pions are more easily detected using these *FC* processes than using the rare top decays $t \rightarrow cl_i l_j$. However, our work provides one possible method to study the signals of the neutral scalar at the *LHC* experiments.

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